"…Euclid alone
Has looked on Beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone."
—Edna St. Vincent Millay

One in Two, Two in One: Mathematics and the Arts

Carla Farsi and Doug Craft
University of Colorado, Boulder and Lakewood, CO

The Taoist yin-yang symbol denoting the complementary union of opposites within the whole.

Mathematics and art are related endeavors? The view of most people is that art and mathematics could not be more different. One is left brain, the other right brain. One is creative, the other analytical. While this prevailing view is a partial truth, what mathematician has not marveled at the beauty of an elegant proof, and what serious artist has not been aware of the importance of form and composition to a successful art work? Both disciplines are creative endeavors with analytical components that are essential elements of contemporary civilization. Here, we hope to make the case that, like the yin/yang symbol, art and mathematics are really one in two, and two in one.

Here, we will discuss some of the ways that mathematics and art are both similar and different; how they express complementary aspects of life and culture; and how they both share a fundamental unity on a deeper level. Mathematics can indeed be defined as the general science of pattern and structure. Because art also involves patterns and structures, art and math relate to each other in many natural ways, which is the basis for this exploration. This exploration has also produced collaborations with artists working with mathematical themes and has coincided with several recent art exhibitions in Colorado. These art exhibitions have been exploring the idea that art and mathematics are intimately related on a deeper level. We will also discuss some recent Colorado art exhibitions that provide good examples of how the collaboration between mathematics and art has found public expression.

Two In One

First, let's examine some of the more obvious differences, or "unparallels" between mathematics and the arts. In math, language and methodologies are standardized and more objective: all mathematics must be expressed in very rigorous language and meet stringent standards of proof and peer review. For example, any mathematician with enough knowledge of the topic under discussion can decide if a derivation is correct or not. Art is much more subjective, less precise, and the criteria for judging the merit of artwork are not uniform. While there are "eternal laws" of form and composition, these are not generally taught or expressed in a highly codified or structured way.

Another important difference between mathematics and the arts is mathematics' high potential for practical applications, especially to science and technology. Mathematical formulas and theories can be applied to create startling new technological innovations and modifications in today's world. For example, the development of the calculus was central to the development of the Industrial Revolution and modern engineering. Mathematics can be molded into serving as an invaluable tool for the dramatic modification of technology. Art does not generally have this function.

One In Two

When considering similarities between mathematics and art, it is clear that both disciplines share a wonderful *creative* aspect. While the external expressions and techniques of art and mathematics could not be more different, the fundamental creativity required to be practitioners is central to both disciplines. Both mathematicians and artists find that our vocations take us away from the mundane world of everyday life, and transform us into grown-up creative children of today's world. And the mysterious and inexplicable contribution of creative inspiration is at the heart of every great work of art and mathematics.

Another concept common to both art and mathematics is *beauty*. Here we are not talking about beauty as the culturally defined idea of pleasing or attractive. By beauty we mean Aquina's term constantia, as harmony with or conformance to the eternal laws of form. For mathematics, this would be the adherence to the logic of the language of mathematics and the emphasis on elegance: that the best proof is the one that is most direct requiring the least assumptions. For art, it would be the acknowledged importance of form and composition; that the best art shows a clear harmony between form and subject.

An example where art and mathematics combine to create harmony would be the artistic use of the Golden Section (ratio, or proportion), called phi, φ, and approximated by the irrational number 1.618034.... Knowledge that φ represents a fundamental structural element of the physical universe has been known since before the dawn of civilization. This ratio occurs throughout nature at all size scales. The Egyptians and Greeks celebrated and applied this sacred metaphorical number, and ancient as well as modern artists have used the Golden Rectangle (see figure below) and φ as a fundamental compositional guide. Notable western practitioners of the Golden Proportion include Leonardo, Dürer, Seurat, Mondrian, Le Corbusier, and Ernst.

The past several decades has seen a renewed interest in the mathematics of the natural world in areas such as minimal surfaces, fractals, symmetries, mirror reflections, chaos theory, and complexity. Numbers and mathematics are intrinsically present in natural objects, and both artists and mathematicians have used these ideas in their respective disciplines.

Mathematical formulas and geometric figures can be aesthetically pleasing in and of themselves. Physicist Wolfgang Pauli said, "It is more important for an equation to be beautiful than exact," suggesting an aesthetic appreciation of what is implied by the mathematical statement of cause and effect. As an example, Einstein's gravitation equation from general relativity, $G_{\mu\upsilon} = 8\pi T_{\mu\upsilon}$, speaks of a universal mystery in elegant form. Similarly, the geometric ornamentation common to Islamic art shows how geometric figures have an intrinsic aesthetic appeal. (See for example the picture below.)

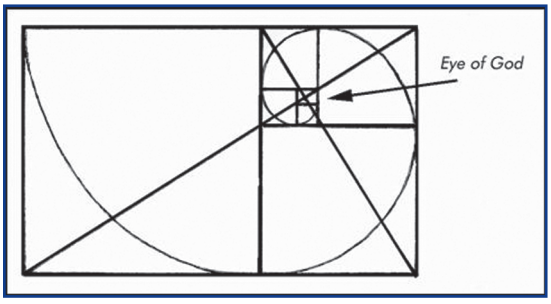
The great power of abstraction/synthesis of art and mathematical knowledge is well known. Mathematical equations can represent behaviors of natural systems in very simple, but meaningful and powerful ways. Further, some forms of equations can also be applied to different phenomena at different size scales, so there is metaphorical implication of physical law expressed through mathematics. For example, wave equations were used in artillery calculations and to describe mechanics of vibrating strings before being adapted by Schröedinger as $\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-U)\psi = 0$ to describe the energy levels in an atom. Diffusion equations can be applied to molecules, or social phenomena like emigration or the spread of ideas.

Similarly, the subject matter of representational art often has a metaphorical or allegorical content. These types of images have multiple layers of meaning and speak to the viewer according to their education and experience. Abstract art also operates on a metaphorical level by eliciting archetypal forms and shapes as well as emotional responses by the application of color, texture, and form. In some cases, such as the drip paintings of Jackson Pollock, the form and composition of abstract art may actually be representational of the fractal structure of the universe.

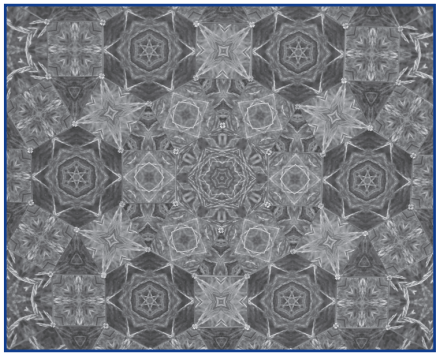
One aspect not yet mentioned that is common to both proper art and mathematics is their deeper purpose to reveal truth about our world and ourselves. Both disciplines strive to discover and represent the unknown and in this process the question of the meaning of our lives is never too far away.

The Colorado Experience–Art and Mathematics Exhibitions

The past 10 years in Colorado has seen a fascinating growth of interest and expression in the interaction of mathematics

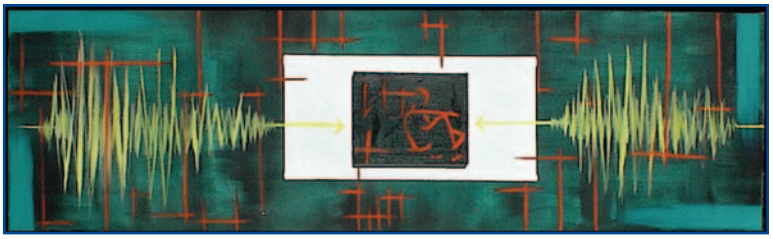


The Golden Rectangle–Image by Doug Craft–shows subdivision by proportional squares that leave subordinate Golden Rectangles. The subdivision converges on the intersection of the diagonal with radical, called the "Eye of God" and provides a framework for a logarithmic spiral.



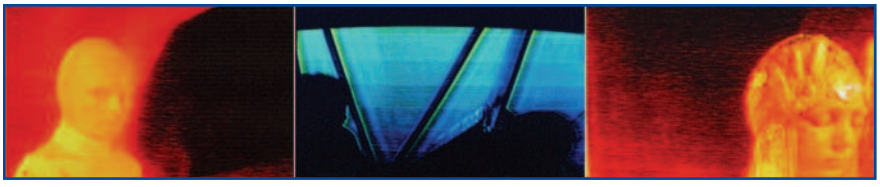
Art inspired by Islamic art–Janet Strickler–Kaleidoscope, 20" x 12", mixed media, 2000.

Carla Farsi–Sun-Moon Lovers, 7-1/2′ x 6-1/2′, oil on canvas, 2003.



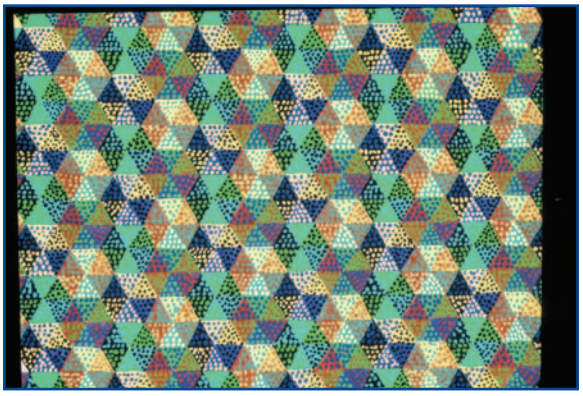
Russ Rockne, Intersection– "(un)Parallels," mixed media, 10" x 36", 2002.

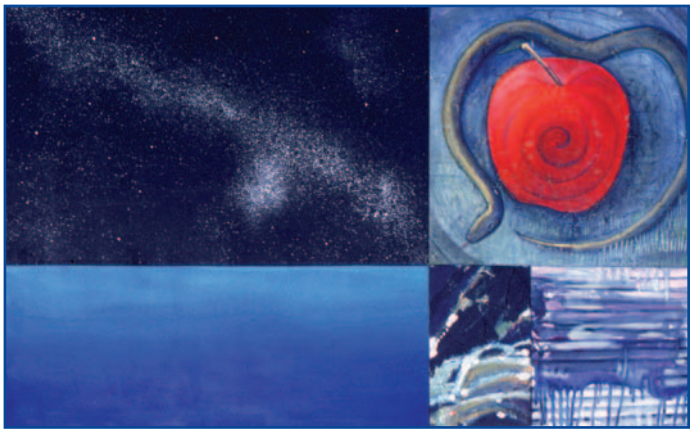
Jerry Malitz–Temple of a Lesser God, in "(un)Parallels," mixed media, 48" x 32" x 1", 1998.



Doug Craft, *I Don't Love You Anymore*, photomontage, 7.5" x 36", framed, 2000.
For more information visit `http://www.DougCraftFineArt.com`.

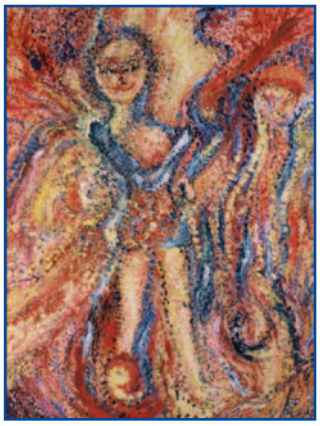
Richard Roth–Painting by numbers, in "(un)Parallels," Acrylic, 36" x 20", 1987.





Doug Craft, Dave Griffin, Rolf Helland, Kent Rucker, *Sacred Rectangle Number 1*, collaborative collage painting from *Sacred Geometry 2003* exhibit, May-June 2003 in Denver, Colorado.

Photos provided by artists.


Carla Farsi–*Self-Portraits*: At one of her 2003 art show openings, golden self-portrait, 8-1/2′ x 6-1/2′, oil on canvas, 2002, and self-standing self-portrait, 6′ x 4′ x 2′, mixed media, 2002.

and art. A small but expanding community of artists, mathematicians, and scientists has formed informal associations through local co-op galleries and universities, and has produced several recent and planned art exhibitions in Colorado:

(Un)Parallels: Mathematics and Art, and Beyond: This exhibit was linked to the class, Mathematics from the Visual Arts, taught by Professor Carla Farsi at the University of Colorado, Boulder. It explored historical links between mathematics and the visual arts, and students were encouraged to creatively express their own views on mathematics, art, and life. The art created by mathematicians, students, and staff, demonstrated that mathematics embodies beauty and harmony as well as being highly creative in a way similar to art.

Sacred Geometry 2003: An exhibit of Golden Rectangle paintings at V-Gallery in Denver May 23 through June 14, 2003 organized by Denver artist Doug Craft. This exhibit featured collaborative paintings with four other local artists exploring the themes of sacred geometry and improvisation. Craft worked with local artists Dave Griffin, Kent Rucker, Roger Rapp, and Rolf Helland to produce five large collage paintings. Each artist independently painted a set of four canvases with dimensions based on consecutive Fibonacci numbers that formed a 55″×34″ golden rectangle. When the individual paintings were completed, the group met and mixed the different proportional canvases to create the five collaged golden rectangles featured in Sacred Geometry 2003.

A Beautiful World: Mathematics and Art: This exhibit at Arapahoe Community College Gallery in Denver explored the theme of the creative relationship between art and math and represented different perspectives on the connection between beauty, art, and mathematics as explored by professionals directly connected to the scientific field, artists working on mathematical themes, and students of scientific and artistic disciplines. The exhibit featured work by Doug Craft, Carla Farsi, Stan Gudder, Jim Johnson, Jerry Malitz, Russ Rockney, Sue Simon, Janet Strickler, and others.

University of Colorado Year in Mathematics and Art: To deepen the investigation and understanding of the relationship between art and mathematics, the CU Departments of Art and Art History and Mathematics, together with the University Memorial Center galleries, designated the year 2005 to be the Special Year in Art and Mathematics. The Special Year will explore the relationship between art and mathematics with an accent on the visual arts through an exciting series of planned events. Besides a variety of visual art exhibits on art/math themes, we are planning an international conference, classes, a series of movies and lectures, concerts, and plays. The cities of Boulder, Denver, and other local arts institutions are also partners in the Special Year activities. These partners include Naropa University, National Center for Athmospheric Research, the Visual Arts Department at the University of Northern Colorado, the Boulder Public Library, the Colorado Gallery at Arapahoe College, and Core New Art Space Gallery and Studio Aiello. This makes our program unique with its first-class interdisciplinary activities and quality partnerships at the local, national, and international level. For more information on the Special Year in Art and Math, please visit the website at `http://math.colorado.edu/Art&Math`.

Further Reading

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